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# REFRACTION EFFECT IN SATELLITE TRACKING

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Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
UNDER GRANT NO. NGR 12-001-045

HAWAII INSTITUTE OF GEOPHYSICS
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## ABSTRACT

*The main environmental problem in tracking a satellite through the atmosphere is in finding the most probable value of the mean refractive index. In this report, the mean refractive index is computed as a four-part model. The troposphere is treated as one altitude range from sea level to 9 km, and the stratosphere is divided into three altitude ranges, 9 to 18, 18 to 27, and 27 to 36 kilometers. At 36 km, the N-value is approximately equal to two and reduces rapidly to zero. By the use of the Essen [5] formula in radio wave applications and the modified Kohlrausch [7] formula in light-wave applications, point-to-point values of the refractive index are computed through these altitude ranges. The polynomial expansion of second order from the basic exponential function is selected as the model, and the curve-fitting adjustments of the computed values are established separately to each altitude range to obtain coefficients A, B, and C.*

*A model based on the U. S. Standard Atmosphere, 1962, [10] is used as the reference to which four sets of actual soundings made in Lihue, Hawaii and Fairbanks, Alaska on February 3 and July, 1966, are compared. The results show that the parabolic adjustment has a very high reliability. In the use of standard atmosphere, the standard error of the mean refractive index through the total altitude range of 0 to 36 km, and at the $70^{o}$ zenith distance, equals only ± 7 mm of distance when radio waves are utilized, and ± 3 mm of distance when light waves are utilized.*

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## 1. Geometry

In this report only the refraction effect affecting the arc-chord correction and the velocity of radio and light waves is discussed, e.g., the problem of distance measurements. In the tracking of satellites various zenith distances should be anticipated. Because of the short distances involved, the various altitude ranges inside the atmosphere are considered as parallels to the flat earth, and the tracking distance S as a function of the altitude range h can be written:

$$S = \frac{h}{\cos \psi} \tag{1}$$

where $\psi$ is the zenith angle.

Corresponding to the zenith angles, $\psi$ = 0°, 30°, 50°, 60°, and 70°, the tracking distances, equivalent to the 9-km altitude range, are S = 9.0, 10.4, 14.0, 18.0, and 26.3 kilometers.

### 1.1 Curvature of the Ray Path

To investigate the significance of the geometric correction to be applied in order to reduce the arc-distance of the ray path to the chord-distance, and taking into account the fact that at different zenith angles the electromagnetic energy propagates through proportionally slightly different atmospheric layers, and thus affects the velocity, the following formula is used [5] ]6]:

$$Q = - \left[B + C(h + k) - \frac{CS^2}{6} \left\{ \frac{1}{R} + B + C(h + k) \right\} \right] \cos \alpha \tag{2}$$

where Q is the curvature of the ray path, R is the curvature radius of the earth in kilometers, S is the tracking distance at various zenith angles in kilometers, h is the elevation of the altitude range in kilometers, $k$ is the elevation of the tracking station in kilometers, $\alpha$ is an angle between the tracking sighting and the horizontal ($\alpha = 90° - \psi$), and B and C are meteorological constants obtained from the NACA moist atmosphere [5]:

$$\begin{aligned} B &= -0.00003670 \\ C &= \phantom{-}0.000001350 \end{aligned} \tag{3}$$

The radius of curvature, $R' = 1/Q$, of the ray path was computed for the zenith angles $\psi = 0°$, $30°$, $50°$, $60°$, and $70°$ and for the altitude range of 0 to 9 km by the assumption that $k = 0$ and $h = 9$ kilometers. The results in kilometers are given in Table I as follows:

TABLE I

| | | | | |
|---|---|---|---|---|
| $\psi = 0°$ | 30° | 50° | 60° | 70° |
| $R' = \infty$ | 81,453 | 53,160 | 47,016 | 43,311 |

The correction of the arc-distance to chord-distance is obtained by the following well-known formula:

$$\Delta S = \frac{S^3}{24\ R'^2} \tag{4}$$

where S is the arc distance of the ray path, and R' is the curvature radius of the ray path.

By selecting the most favorable zenith distance, $\psi = 70°$, for comparison, and by applying $S = 26.3$ km and $R' = 43,311$ km into Eq. (4), the correction will be $\Delta S = 0.4$ mm, which in all practical and even theoretical applications can be neglected.

## 1.2 Model of the Refractive Index as a Function of the Atmosphere

Models of the refractive index can be established on the basis of a standard atmosphere or an actual atmosphere based on meteorological soundings. There are several approaches in forming the model from standard atmosphere: four of them are discussed here.

Henriksen [2] (Army Map Service) expresses the N-value, which is equivalent to the sixth decimal of the refractive index [$N = (n - 1) \cdot 10^6$], from zero altitude to the upper limit of the atmosphere by the exponential curve:

$$N = 325 \cdot e^{-.1312 \cdot h \cdot 10^{-3}} \tag{5}$$

where the sea-level value is assumed to be $n_o = 1.000325$, and h is the height above sea level in meters.

Bean [1] [2] (The National Bureau of Standards) expresses the refractive index through atmosphere as a three-part model. He assumes linear decrease in N for the first kilometer above the station. From $(h_s + 1)$ to 9 km, he uses the exponential function:

$$N = N_1 \cdot e^{-c(h - h_s - 1)} \tag{6}$$

where

$$c = \frac{1}{8 - h_s} \cdot \ln \frac{N_1}{105}$$

and $h_s$ is the height of the station in kilometers. Above 9 km, the following exponential formula is utilized:

$$N = 105 \cdot e^{-.1424\ (h - 9)} \tag{7}$$

Jacobsen [5] (U. S. Air Force) developed a formula which took into consideration also the curvature of the ray path, Q, the curvature of the earth, 1/R, and the slope distance, S, of the tracking at large zenith angles, as follows:

$$n = 1 + A + \frac{B(h + k)}{2} + \frac{C}{3}\,[(h + k)^2 - hk] - \frac{S^2}{12}\,[\,\frac{1}{R} - Q\,]$$
$$[B + C(h + k) - \frac{CS^2}{10}\,(\,\frac{1}{R} - Q\,)] \tag{8}$$

where A = 0.000321. B and C are the same as in (3).

To investigate the effect of the slope propagation of electromagnetic energy in the refractive index at various zenith angles through the basic altitude range from 0 to 9 km, the following values were applied to Eq. (8):

$$\begin{array}{llll} A = & 0.000321 & R = 6371 \text{ kilometers} & \\ B = & -0.00003670 & h = 9.0 \text{ kilometers} & (9) \\ C = & 0.000001350 & k = 0 & \end{array}$$

R'-values were taken from Table I, and the results are given in Table II as follows:

TABLE II

| | | | | |
|---|---|---|---|---|
| $\psi$ = 0° | 30° | 50° | 60° | 70° |
| S = 9.0 km | 10.4 km | 14.0 km | 18.0 km | 26.3 km |
| n = 1.00019233 | 1.00019233 | 1.00019236 | 1.00019239 | 1.00019249 |

The largest difference in n between the zenith angles of 0° and 70° is only $\Delta n = 1.6 \cdot 10^{-7}$, which at the distance of S = 26.3 km is equivalent to 4.2 millimeters. This suggests strongly that at any practical application the effect of the sloping distance in the refractive index can be neglected and that the mean refractive index can be computed on the basis of the vertical propagation of atmospheric parameters.

In the Jacobsen formula, Eq. (8), several approximations were anticipated. To investigate the validity of these approximations, a vertical propagation of the electromagnetic waves can be assumed from sea level. Under this condition k = 0, S = h, and Q = 0, and Eq. (8) can be rewritten as follows:

$$n = 1 + A + \frac{B}{2} h + \frac{C}{3} h^2 + dn \tag{10}$$

where

$$dn = - \left[ \frac{h^2 B}{12R} + \frac{h^3 C}{12R} - \frac{h^4 C}{120R^2} \right] \tag{11}$$

By the application of the values (9) into Eq. (11), dn yields the following magnitude, dn = 0.00000003, or $3 \cdot 10^{-8}$ of the distance. At the most unfavorable slope distance of 26.3 km, dn = 0.8 mm, which is completely negligible, and Eq. (10) minus dn thus represents the mean refractive index between sea level and an altitude of h.

Because of the development of a polynom of second order only, the Jacobsen formula was originally used for measuring distances to aircrafts not exceeding the altitude of about 10 kilometers.

The author presents the refractive index through the atmosphere as a four-part model. The second order parabolic development of the basic exponential function is applied as one altitude range through the troposphere, and as three separate altitude ranges through the stratosphere.

The second order parabola:

$$n = 1 + A + Bh + Ch^2 \tag{12}$$

is valid only for the point-to-point determination of the refractive index when the coefficients A, B, and C and the altitude h is known. In the utilization of refractive index through the atmosphere for satellite tracking, the best-fitting mean refractive index obtained through various altitude ranges is more significant than the point-to-point values, and therefore, instead of Eq. (12), the mean value should be used as obtained from the following expression:

$$\bar{n} = \frac{1}{h} \int_o^h n \, dh \tag{13}$$

or

$$\bar{n} = A + \frac{h}{2} B + \frac{h^2}{3} C \tag{14}$$

which is practically the same as Eq. (10).

Also, the most realistic criterion in investigating the reliability, or validity, of such mean refractive index is to compute the standard error of function (14). According to the general error propagation it will be:

$$m_n = \pm\mu \sqrt{[FF]} \tag{15}$$

where μ is the standard error of the weight unit, and [FF] is the weight number of function (14) obtained as follows:

$$[FF] = f'^2_A\ [\alpha\alpha] + 2f'_A f'_B\ [\alpha\beta] + 2f'_A f'_C\ [\alpha\gamma] + f'^2_B\ [\beta\beta] + 2f'_B f'_C\ [\beta\gamma] + f'^2_C\ [\gamma\gamma] \tag{16}$$

[αα], [ββ], and [γγ] are the weight numbers and [αβ], [αγ], and [βγ] are the correlation numbers of unknowns A, B, and C. Partial derivatives for the unknowns are denoted by $f'_A$, $f'_B$, and $f'_C$, correspondingly. When applied to Eq. (14), the following expression for [FF] can be written:

$$[FF] = [\alpha\alpha] + h[\alpha\beta] + \frac{2}{3}\,h^2[\alpha\gamma] + \frac{h^2}{4}\,[\beta\beta] + \frac{h^3}{3}\,[\beta\gamma] + \frac{h^4}{9}\,[\gamma\gamma] \tag{17}$$

## 2. Point-to-Point Determination of the Refractive Index for the Radio Waves.

To determine the coefficients of any model by the least-squares curve-fitting method, observed values of the refractive index must first be computed. The three formulas which follow are commonly used in the determination of the refractive index as a function of the atmospheric parameters--temperature, pressure, and humidity:

Essen Formula:

$$N = (n - 1) \cdot 10^6 = \frac{77.62}{T}\,P - \left(\frac{12.92}{T} - \frac{37.19 \cdot 10^4}{T^2}\right) e \tag{18}$$

Essen-Froome Formula:

$$N = (n - 1) \cdot 10^6 = \frac{103.49}{T} (P - e) + \frac{86.26}{T} (1 + \frac{5748}{T}) e \qquad (19)$$

Smith-Weintraub Formula:

$$N = (n - 1) \cdot 10^6 = \frac{77.6}{T} (P + 4.81 \cdot 10^3 \cdot \frac{e}{T}) \qquad (20)$$

In the Essen and Smith-Weintraub formulas, T is the temperature in Kelvin units, P is the total pressure in millibars, and e is the partial pressure of water vapor in millibars. At the 1960 meeting of the International Union of Geodesy and Geophysics [4] it was recommended that the Essen-Froome formula be used to achieve uniformity in calculations of electronic distance measurements. It should be noted, however, that the Essen-Froome formula is actually the Essen formula obtained by rearranging terms and stating pressures in millimeters of mercury instead of millibars.

Essen and Smith-Weintraub formulas were compared by applying to both formulas T and P values from the U. S. Standard Atmosphere 1962 [10] and e value from Eq. (30) as follows:

TABLE III

| Altitude, km | Essen N-value | Smith-Weintraub N-value | Difference, $\Delta N$ |
|---|---|---|---|
| 0 | 321.0 | 321.6 | -0.6 |
| 1 | 285.6 | 286.1 | -0.5 |
| 2 | 253.0 | 253.3 | -0.3 |
| 3 | 223.0 | 223.2 | -0.2 |
| 4 | 195.7 | 195.9 | -0.2 |
| 5 | 171.1 | 171.2 | -0.1 |

Consequently, the use of either formula is justified. In this report, the Essen formula has been utilized because, in actual soundings, the total pressure is given in millibars and the partial pressure of water vapor is given either as the dew point temperature or as the relative humidity. In both cases it is easy to convert them to millibars by using the Smithsonian Meteorological Tables.

## 2.1 Sensitivity of the Atmospheric Parameters

To evaluate the error in n caused by the uncertainty in observing the parameters T, P, and e, the Essen formula is differentiated as follows:

If

$$N = (n - 1) \cdot 10^6 = \frac{K}{T} P - \left( \frac{L}{T} - \frac{M \cdot 10^4}{T^2} \right) e \tag{21}$$

where K = 77.62, L = 12.92, and M = 37.19, it is obtained:

$$dN_T = \left[ - \frac{KP}{T^2} + \left( \frac{L}{T^2} - \frac{2 \cdot M \cdot 10^4}{T^3} \right) e \right] dT$$

$$dN_P = \frac{K}{T} dP \tag{22}$$

$$dN_e = \left( - \frac{L}{T} + \frac{M \cdot 10^4}{T^2} \right) de$$

If the criterion for the sensitivity of observing the parameters is set forth to be 1 ppm, and K, P, and e are taken from the U. S. Standard Atmosphere and from Eq. (30) to correspond to the altitude of 4.5 km, the following values are applied to Eq. (22):

$$T = 258^\circ.9$$
$$P = 577 \text{ mb} \qquad (23)$$
$$e = 1.84 \text{ mb}$$

The standard error of each individual parameter to cause an error of 1 ppm in n thus will be:

$$m_T = \pm 1^\circ.3$$
$$m_P = \pm 3.3 \text{ mb} \qquad (24)$$
$$m_e = \pm 0.2 \text{ mb}$$

From (24) it is clearly seen that, in tracking satellites by the use of radio waves, the determination of the humidity is the most critical phase in the operation.

## 3. Standard Atmosphere

To evaluate the magnitude of errors made in obtaining the meteorological parameters from a standard atmosphere instead of a local atmosphere based on actual soundings, the U. S. Standard Atmosphere, 1962, [10] has been selected as the basis for this analysis. This atmosphere is fundamentally defined in terms of an ideal air obeying the perfect gas law. It is based upon accepted standard values of air density, temperature, and pressure at sea level as follows:

$$\text{Density} \quad \rho_o = 1.2250 \text{ kg/m}^3$$
$$\text{Temperature} \quad T_o = 288^\circ.15 \text{ K} \qquad (25)$$
$$\text{Pressure} \quad P_o = 1013.25 \text{ millibars}$$

The density for tabulated altitudes is obtained from the following expression:

$$p = \frac{M \cdot P}{R \cdot T} \tag{26}$$

Where M = 28.9644 is the molecular weight of air and is considered as a constant up to an altitude of 90 km, P is the pressure, R is the universal gas constant, and T is the absolute temperature.

The temperature variation is defined as a series of connected segments, linear in latitude. The general formula for each linear segment is:

$$T = T_n + L(H + H_n) \tag{27}$$

where L is the gradient of the temperature with altitude dT/dH. Between $H_n = 0$ and H = 11 km, L = -6°.5 K/1 km; between $H_n = 11$ km and H = 20 km, L = 0°.0 K/1 km; between $H_n = 20$ km and H = 32 km, L = +1°.0 K/1 km; and between $H_n = 32$ km and H = 47 km, L = +2°.8 K/1 km.

Within an atmospheric layer throughout which T is a linear function of H, the pressure is computed by the following formula when $L \neq 0$:

$$\mathrm{Log}_e\, P = \mathrm{Log}_e\, P_n - \frac{g_o M}{L \cdot R}\, \mathrm{Log}_e \frac{L(H - H_n) + T}{T} \tag{28}$$

where $g_o = 9.80665\ \mathrm{m/sec}^2$.

If L = 0, the following expression for P is used:

$$\mathrm{Log}_e\, P = \mathrm{Log}_e\, P_n - \frac{g_o M}{R \cdot T} (H - H_n) \tag{29}$$

Partial pressure of water vapor is not readily obtainable from the U. S. Standard Atmosphere. For its determination, the authors established the following procedure [8]: Coefficients A, B, and C from (9) were applied to Eq. (12) which then was solved simultaneously with the Essen formula, Eq. (21) for e:

$$e = \frac{T^2 \left( \frac{K}{T} P \quad 10^{-6} - A - Bh - Ch^2 \right)}{(TL - M \quad 10^4) \quad 10^{-6}} \tag{30}$$

All altitude values, H, in Eqs. (27) - (29) are geopotential altitudes, while h in Eq. (30) is geometric altitude.

## 3.1 Strength of the Parabolic Assumption

To investigate the strength of the parabolic assumption, 74 values of T and P were taken from the U. S. Standard Atmosphere, together with the e-values from Eq. (30) at intervals of 0.5 km from sea level to an altitude of 36.5 kilometers. By the use of the Essen formula, the N-values were computed, Table IV.

TABLE IV

| No. | Geom. Alt., km | T, °K | P, mb | e, mb | N |
|---|---|---|---|---|---|
| 1 | 0 | 288.2 | 1013.0 | 10.87 | 321.0 |
| 2 | 0.5 | 284.9 | 955.0 | 9.43 | 302.9 |
| 3 | 1.0 | 281.7 | 899.0 | 8.18 | 285.7 |
| 4 | 1.5 | 278.4 | 846.0 | 6.96 | 268.9 |
| 5 | 2.0 | 275.2 | 795.0 | 5.91 | 252.9 |
| 6 | 2.5 | 271.9 | 747.0 | 4.90 | 237.7 |
| 7 | 3.0 | 268.7 | 701.0 | 4.02 | 223.0 |
| 8 | 3.5 | 265.4 | 658.0 | 3.17 | 209.0 |
| 9 | 4.0 | 262.2 | 617.0 | 2.44 | 195.7 |
| 10 | 4.5 | 258.9 | 577.0 | 1.84 | 183.1 |
| 11 | 5.0 | 255.7 | 540.0 | 1.28 | 171.1 |

TABLE IV (continued)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 5.5 | 252.4 | 505.0 | 0.79 | 159.9 |
| 13 | 6.0 | 249.2 | 472.0 | 0.39 | 149.3 |
| 14 | 6.5 | 245.9 | 441.0 | 0.03 | 139.4 |
| 15 | 7.0 | 242.7 | 411.0 | 0 | 131.4 |
| 16 | 7.5 | 239.5 | 383.0 | 0 | 124.1 |
| 17 | 8.0 | 236.2 | 357.0 | 0 | 117.3 |
| 18 | 8.5 | 232.9 | 332.0 | 0 | 110.6 |
| 19 | 9.0 | 229.7 | 308.0 | 0 | 104.1 |
| 20 | 9.5 | 226.5 | 286.0 | 0 | 98.0 |
| 21 | 10.0 | 223.3 | 265.0 | 0 | 92.1 |
| 22 | 10.5 | 220.0 | 245.0 | 0 | 86.5 |
| 23 | 11.0 | 216.7 | 227.0 | 0 | 81.3 |
| 24 | 11.5 | 216.7 | 210.0 | 0 | 75.2 |
| 25 | 12.0 | 216.7 | 194.0 | 0 | 69.5 |
| 26 | 12.5 | 216.7 | 179.0 | 0 | 64.1 |
| 27 | 13.0 | 216.7 | 166.0 | 0 | 59.5 |
| 28 | 13.5 | 216.7 | 153.0 | 0 | 54.8 |
| 29 | 14.0 | 216.7 | 142.0 | 0 | 50.9 |
| 30 | 14.5 | 216.7 | 131.0 | 0 | 46.9 |
| 31 | 15.0 | 216.7 | 121.0 | 0 | 43.3 |
| 32 | 15.5 | 216.7 | 112.0 | 0 | 40.1 |
| 33 | 16.0 | 216.7 | 104.0 | 0 | 37.3 |
| 34 | 16.5 | 216.7 | 96.0 | 0 | 34.3 |
| 35 | 17.0 | 216.7 | 88.0 | 0 | 31.5 |
| 36 | 17.5 | 216.7 | 82.0 | 0 | 29.4 |
| 37 | 18.0 | 216.7 | 76.0 | 0 | 27.2 |
| 38 | 18.5 | 216.7 | 70.0 | 0 | 25.1 |
| 39 | 19.0 | 216.7 | 65.0 | 0 | 23.3 |
| 40 | 19.5 | 216.7 | 60.0 | 0 | 21.5 |
| 41 | 20.0 | 216.7 | 55.0 | 0 | 19.7 |
| 42 | 20.5 | 217.1 | 51.0 | 0 | 18.2 |
| 43 | 21.0 | 217.6 | 47.0 | 0 | 16.8 |
| 44 | 21.5 | 218.1 | 44.0 | 0 | 15.7 |

TABLE IV (continued)

| | | | | | |
|---|---|---|---|---|---|
| 45 | 22.0 | 218.6 | 40.0 | 0 | 14.2 |
| 46 | 22.5 | 219.1 | 37.0 | 0 | 13.1 |
| 47 | 23.0 | 219.7 | 35.0 | 0 | 12.4 |
| 48 | 23.5 | 220.1 | 32.0 | 0 | 11.3 |
| 49 | 24.0 | 220.6 | 30.0 | 0 | 10.6 |
| 50 | 24.5 | 221.1 | 28.0 | 0 | 9.8 |
| 51 | 25.0 | 221.6 | 25.0 | 0 | 8.8 |
| 52 | 25.5 | 222.0 | 23.0 | 0 | 8.0 |
| 53 | 26.0 | 222.5 | 22.0 | 0 | 7.7 |
| 54 | 26.5 | 223.0 | 20.0 | 0 | 7.0 |
| 55 | 27.0 | 223.5 | 19.0 | 0 | 6.6 |
| 56 | 27.5 | 224.0 | 17.0 | 0 | 5.9 |
| 57 | 28.0 | 224.5 | 16.0 | 0 | 5.5 |
| 58 | 28.5 | 225.0 | 15.0 | 0 | 5.2 |
| 59 | 29.0 | 225.5 | 14.0 | 0 | 4.8 |
| 60 | 29.5 | 226.0 | 13.0 | 0 | 4.4 |
| 61 | 30.0 | 226.5 | 12.0 | 0 | 4.1 |
| 62 | 30.5 | 227.0 | 11.1 | 0 | 3.8 |
| 63 | 31.0 | 227.5 | 10.3 | 0 | 3.5 |
| 64 | 31.5 | 228.0 | 9.6 | 0 | 3.3 |
| 65 | 32.0 | 228.5 | 8.9 | 0 | 3.0 |
| 66 | 32.5 | 229.6 | 8.2 | 0 | 2.8 |
| 67 | 33.0 | 231.0 | 7.7 | 0 | 2.6 |
| 68 | 33.5 | 232.3 | 7.3 | 0 | 2.4 |
| 69 | 34.0 | 233.7 | 6.6 | 0 | 2.2 |
| 70 | 34.5 | 234.6 | 6.2 | 0 | 2.0 |
| 71 | 35.0 | 236.5 | 5.7 | 0 | 1.9 |
| 72 | 35.5 | 237.9 | 5.4 | 0 | 1.8 |
| 73 | 36.0 | 239.3 | 5.0 | 0 | 1.6 |
| 74 | 36.5 | 240.6 | 4.6 | 0 | 1.5 |

The computed N-values were adjusted separately for the four altitude ranges, 0 to 9, 9 to 18, 18 to 27, and 27 to 36 kilometers. Curve-fitting adjustment of Eq. (12) yielded the following constants, A, B, and C, given in Table V.

TABLE V

| Altitude Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0003212 | -0.000037141 | 0.0000014370 |
| 9 - 18 | 0.0001062 | -0.000013897 | 0.0000005693 |
| 18 - 27 | 0.0000267 | -0.000003786 | 0.0000001766 |
| 27 - 36 | 0.0000064 | -0.000000870 | 0.0000000380 |

The mean refractive indexes $\bar{n}$ and their standard errors in N-units for each altitude range are given in Table VI. In Table VII, residuals after the adjustment are given at intervals of 1 km from 0 to 15 km, and at intervals of 5 km from 15 to 35 kilometers. E denotes the Essen point-to-point values, L denotes the values read from the adjusted parabolas, and L, $\Delta N$ denotes their differences.

TABLE VI

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001929 | ± 0.18 |
| 9 - 18 | 1.0000591 | ± 0.09 |
| 18 - 27 | 1.0000144 | ± 0.05 |
| 27 - 36 | 1.0000036 | ± 0.02 |

TABLE VII

| Altitude, km | E, N-Value | L, N-Value | L, ΔN-Value | Altitude, km | E, N-Value | L, N-Value | L, ΔN-Value |
|---|---|---|---|---|---|---|---|
| 0 | 321.0 | 321.2 | -0.2 | 11 | 81.3 | 80.7 | +0.6 |
| 1 | 285.7 | 285.5 | +0.2 | 12 | 69.5 | 69.6 | -0.1 |
| 2 | 252.9 | 252.6 | +0.3 | 13 | 59.5 | 59.7 | -0.2 |
| 3 | 223.0 | 222.7 | +0.3 | 14 | 50.9 | 50.9 | 0 |
| 4 | 195.7 | 195.6 | +0.1 | 15 | 43.3 | 43.3 | 0 |
| 5 | 171.1 | 171.4 | -0.3 | 20 | 19.7 | 19.8 | -0.1 |
| 6 | 149.3 | 150.2 | -0.9 | 25 | 8.8 | 8.9 | -0.1 |
| 7 | 131.4 | 131.6 | -0.2 | 30 | 4.1 | 4.2 | -0.1 |
| 8 | 117.3 | 116.1 | +1.2 | 35 | 1.9 | 1.8 | +0.1 |
| 9 | 104.1 | 103.3 | +0.8 | | | | |
| 10 | 92.1 | 92.9 | -0.8 | | | | |

The standard errors from Table VI were converted in meters of distance corresponding to the zenith angles of $\psi$ = 70°, 60° 50°, 30°, and 0°. Since each altitude range was adjusted independently, the total cumulative standard error for the entire altitude range from sea level to 36 km was obtained by the normal, quadratic error cumulation. The results are given in Table VIII.

TABLE VIII

| Altitude Range, km | Zenith Angle | | | | |
|---|---|---|---|---|---|
| | 70° | 60° | 50° | 30° | 0° |
| 0 - 9 | ±0.005 | +0.003 | ±0.002 | ±0.002 | ±0.002 |
| 9 - 18 | 0.003 | 0.002 | 0.001 | 0.001 | 0.001 |
| 18 - 27 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 |
| 27 - 36 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0 - 36 | 0.007 | 0.004 | 0.002 | 0.002 | 0.002 |

The largest standard error at the poorest zenith angle, $\psi = 70°$, and through the most vulnerable altitude range, 0 to 9 km, where moisture is present, is only ± 5 millimeters. This, together with the small residuals in Table VII prove the parabolic curve-fitting adjustment to be very reliable.

## 4. Model of the Refractive Index of Radio Waves from Actual Meteorological Soundings

The effect of the use of actual meteorological soundings in the vicinity of the tracking station was investigated by selecting four widely separated climatological samples as reference. The sounding data were obtained from the U. S. Weather Bureau stations at Lihue, February 3, 1966; Lihue, July 2, 1966; Fairbanks, February 3, 1966; Fairbanks, July 2, 1966. All soundings were made at 00.00 Greenwich time.

These samples represented hot and humid, cool and dry, and cold and dry climatological conditions, with the rather anomalous temperature distribution as shown in Table IX, given in centigrade:

TABLE IX

| Altitude, km | Lihue July | Lihue February | Fairbanks July | Fairbanks February | U. S. Standard |
|---|---|---|---|---|---|
| 0.5 | 21 | 17 | 23 | - 8 | 12 |
| 5.0 | 0 | - 5 | -15 | -30 | -17 |
| 10.0 | -38 | -35 | -43 | -54 | -50 |
| 15.0 | -69 | -71 | -45 | -46 | -57 |
| 17.0 | -69 | -75 | -45 | -47 | -57 |

All sounding altitudes were given as geopotential heights, which were converted into geometric heights by the use of the Smithsonian

Meteorological Tables, based on the following formula:

$$H = \frac{M \cdot Z}{\left(\frac{g_\phi \cdot M}{9.8}\right) - Z} \tag{31}$$

where Z is the geopotential height, $g_\phi$ is the actual acceleration of gravity at latitude $\phi$, and M is the meridian curvature radius at that latitude.

After this conversion, actual soundings were applied to the Essen formula, Eq. (18) and are presented in tables X - XIII. Curve-fitting adjustment of Essen values from these tables into the parabola (12) yielded the following constants, A, B, and C given in tables XIV - XVII. The mean refractive indexes through the four altitude ranges and their standard errors in N-units are presented in tables XVIII - XXI. These standard errors were converted to correspond to standard errors in distance, in meters, for the zenith angles $\psi$ = 0°, 30°, 50°, 60°, and 70° and for the altitude ranges of 0 to 9, 9 to 18, 18 to 27, and 27 to 36 km. Also, the constant shift, which occurs in the tracking of satellites when the mean refractive index is taken from the U. S. Standard Atmosphere, instead of from the local atmosphere based on soundings, was computed. It was obtained as the difference of the mean refractive indexes from tables XVIII - XXI and those from Table VI, in meters of distance. The results are given in tables XXII - XXV.

TABLE X

Lihue, Hawaii, February 3, 1966

| No. | Geom. Alt., km | T,°K | P, mb | e, mb | N |
|---|---|---|---|---|---|
| 1 | 0.036 | 294.7 | 1009 | 15.83 | 332.9 |
| 2 | 0.114 | 293.9 | 1000 | 12.36 | 316.8 |
| 3 | 0.552 | 289.7 | 950 | 9.50 | 296.2 |
| 4 | 1.012 | 285.3 | 900 | 9.30 | 286.9 |

TABLE X (continued)

| | | | | | |
|---|---|---|---|---|---|
| 5 | 1.490 | 283.6 | 850 | 4.47 | 253.1 |
| 6 | 1.994 | 282.9 | 800 | 2.80 | 232.4 |
| 7 | 2.505 | 281.2 | 750 | 2.17 | 217.1 |
| 8 | 3.099 | 279.0 | 700 | 1.87 | 203.6 |
| 9 | 3.704 | 276.0 | 650 | 1.51 | 190.1 |
| 10 | 4.348 | 272.7 | 600 | 1.20 | 176.7 |
| 11 | 5.031 | 268.0 | 550 | 0.88 | 163.8 |
| 12 | 5.785 | 262.0 | 500 | 0.58 | 151.2 |
| 13 | 6.575 | 257.0 | 450 | 0.40 | 138.1 |
| 14 | 7.472 | 251.6 | 400 | 0.22 | 124.7 |
| 15 | 8.451 | 247.9 | 350 | 0.17 | 110.6 |
| 16 | 9.564 | 242.3 | 300 | 0.10 | 96.7 |
| 17 | 10.839 | 234.0 | 250 | 0.05 | 83.3 |
| 18 | 12.335 | 222.5 | 200 | 0 | 69.8 |
| 19 | 13.202 | 216.4 | 175 | 0 | 62.8 |
| 20 | 14.167 | 209.7 | 150 | 0 | 55.5 |
| 21 | 15.273 | 201.9 | 125 | 0 | 48.1 |
| 22 | 16.580 | 197.9 | 100 | 0 | 39.2 |
| 23 | 17.884 | 197.8 | 80 | 0 | 31.3 |
| 24 | 18.664 | 198.5 | 70 | 0 | 27.4 |
| 25 | 19.582 | 206.0 | 60 | 0 | 22.6 |
| 26 | 20.695 | 208.0 | 50 | 0 | 18.7 |
| 27 | 22.083 | 212.0 | 40 | 0 | 14.6 |
| 28 | 23.915 | 217.1 | 30 | 0 | 10.7 |
| 29 | 25.130 | 220.0 | 25 | 0 | 8.8 |
| 30 | 26.450 | 221.7 | 20 | 0 | 7.0 |
| 31 | 28.442 | 222.6 | 15 | 0 | 5.2 |
| 32 | 31.137 | 228.1 | 10 | 0 | 3.4 |
| 33 | 33.540 | 230.0 | 7 | 0 | 2.3 |

TABLE XI

Lihue, Hawaii, July 2, 1966

| No. | Geom. Alt., km | T,°K | P, mb | e, mb | N |
|---|---|---|---|---|---|
| 1 | 0.036 | 300.0 | 1013 | 24.42 | 362.0 |
| 2 | 0.150 | 296.7 | 1000 | 18.17 | 337.6 |
| 3 | 0.591 | 292.9 | 950 | 15.79 | 319.5 |
| 4 | 1.062 | 289.3 | 900 | 12.78 | 297.7 |
| 5 | 1.547 | 286.6 | 850 | 10.43 | 276.9 |
| 6 | 2.055 | 284.7 | 800 | 7.42 | 251.8 |
| 7 | 2.594 | 281.8 | 750 | 6.68 | 237.6 |
| 8 | 3.170 | 284.0 | 700 | 1.97 | 200.3 |
| 9 | 3.777 | 281.9 | 650 | 1.82 | 187.4 |
| 10 | 4.446 | 277.2 | 600 | 1.31 | 174.3 |
| 11 | 5.131 | 272.0 | 550 | 0.91 | 161.5 |
| 12 | 5.903 | 266.3 | 500 | 0.63 | 149.0 |
| 13 | 6.717 | 260.0 | 450 | 0.38 | 136.4 |
| 14 | 7.612 | 254.7 | 400 | 0.25 | 123.3 |
| 15 | 8.597 | 247.2 | 350 | 0.14 | 110.7 |
| 16 | 9.699 | 238.7 | 300 | 0.07 | 98.0 |
| 17 | 10.960 | 231.1 | 250 | 0 | 84.0 |
| 18 | 12.447 | 220.0 | 200 | 0 | 70.6 |
| 19 | 13.300 | 212.3 | 175 | 0 | 64.0 |
| 20 | 14.253 | 206.4 | 150 | 0 | 56.4 |
| 21 | 15.344 | 203.0 | 125 | 0 | 47.8 |
| 22 | 16.689 | 203.6 | 100 | 0 | 38.1 |
| 23 | 18.040 | 207.2 | 80 | 0 | 30.0 |
| 24 | 18.855 | 208.4 | 70 | 0 | 26.1 |
| 25 | 19.805 | 209.9 | 60 | 0 | 22.2 |
| 26 | 20.940 | 211.7 | 50 | 0 | 18.3 |
| 27 | 22.347 | 215.5 | 40 | 0 | 14.4 |
| 28 | 24.202 | 220.8 | 30 | 0 | 10.5 |
| 29 | 25.406 | 225.9 | 25 | 0 | 8.6 |

TABLE XI (continued)

| | | | | | |
|---|---|---|---|---|---|
| 30 | 26.900 | 228.0 | 20 | 0 | 6.8 |
| 31 | 28.850 | 230.8 | 15 | 0 | 5.0 |
| 32 | 31.642 | 236.8 | 10 | 0 | 3.3 |
| 33 | 34.165 | 239.3 | 7 | 0 | 2.3 |
| 34 | 36.565 | 241.8 | 5 | 0 | 1.6 |

TABLE XII

Fairbanks, Alaska, February 3, 1966

| No. | Geom. Alt., km | T,°K | P, mb | e, mb | N |
|---|---|---|---|---|---|
| 1 | 0.146 | 254.1 | 1000 | 1.14 | 312.0 |
| 2 | 0.539 | 266.6 | 950 | 2.65 | 290.3 |
| 3 | 0.960 | 267.9 | 900 | 2.17 | 271.9 |
| 4 | 1.406 | 265.6 | 850 | 1.65 | 257.0 |
| 5 | 1.877 | 263.3 | 800 | 1.17 | 242.1 |
| 6 | 2.376 | 259.9 | 750 | 0.85 | 228.6 |
| 7 | 2,893 | 255.3 | 700 | 0.73 | 216.9 |
| 8 | 3.446 | 250.8 | 650 | 0.84 | 206.1 |
| 9 | 4.026 | 247.6 | 600 | 0.30 | 189.9 |
| 10 | 4.658 | 244.5 | 550 | 0.21 | 175.9 |
| 11 | 5.331 | 240.4 | 500 | 0.13 | 162.3 |
| 12 | 6.056 | 235.2 | 450 | 0.08 | 149.0 |
| 13 | 6.871 | 230.0 | 400 | 0 | 135.0 |
| 14 | 7.759 | 224.6 | 350 | 0 | 121.0 |
| 15 | 8.758 | 218.4 | 300 | 0 | 106.6 |
| 16 | 9.917 | 217.6 | 250 | 0 | 89.2 |
| 17 | 11.365 | 223.1 | 200 | 0 | 69.6 |
| 18 | 12.240 | 224.8 | 175 | 0 | 60.4 |
| 19 | 13.261 | 227.7 | 150 | 0 | 51.1 |

TABLE XII (continued)

| | | | | | |
|---|---|---|---|---|---|
| 20 | 13.261 | 227.7 | 150 | 0 | 51.1 |
| 21 | 15.964 | 226.2 | 100 | 0 | 34.3 |
| 22 | 17.442 | 225.7 | 80 | 0 | 27.5 |
| 23 | 18.324 | 225.4 | 70 | 0 | 24.1 |
| 24 | 19.346 | 225.7 | 60 | 0 | 20.6 |
| 25 | 20.556 | 225.7 | 50 | 0 | 17.2 |
| 26 | 22.030 | 226.1 | 40 | 0 | 13.7 |
| 27 | 23.955 | 227.1 | 30 | 0 | 10.3 |
| 28 | 25.173 | 227.6 | 25 | 0 | 8.5 |
| 29 | 26.672 | 228.1 | 20 | 0 | 6.8 |
| 30 | 28.612 | 228.8 | 15 | 0 | 5.1 |
| 31 | 31.362 | 231.3 | 10 | 0 | 3.4 |

TABLE XIII

Fairbanks, Alaska, July 2, 1966

| No. | Geom. Alt., km | T,°K | P, mb | e, mb | N |
|---|---|---|---|---|---|
| 1 | 0.499 | 296.5 | 950 | 5.21 | 270.5 |
| 2 | 0.961 | 291.6 | 900 | 4.07 | 257.2 |
| 3 | 1.445 | 286.4 | 850 | 3.07 | 244.1 |
| 4 | 1.947 | 281.3 | 800 | 2.73 | 233.5 |
| 5 | 2.464 | 276.1 | 750 | 2.29 | 221.9 |
| 6 | 3.027 | 271.7 | 700 | 1.17 | 205.8 |
| 7 | 3.615 | 269.0 | 650 | 0.73 | 191.3 |
| 8 | 4.242 | 264.1 | 600 | 0.50 | 178.9 |
| 9 | 4.900 | 258.3 | 550 | 0.33 | 167.1 |
| 10 | 5.626 | 253.8 | 500 | 0.24 | 154.3 |
| 11 | 6.390 | 251.6 | 450 | 0.21 | 140.0 |
| 12 | 7.263 | 246.2 | 400 | 0.13 | 126.9 |

TABLE XIII (continued)

| | | | | | |
|---|---|---|---|---|---|
| 13 | 8.221 | 241.2 | 350 | 0.09 | 113.2 |
| 14 | 9.295 | 233.9 | 300 | 0.05 | 99.9 |
| 15 | 10.520 | 225.0 | 250 | 0 | 86.2 |
| 16 | 11.973 | 218.8 | 200 | 0 | 71.0 |
| 17 | 12.837 | 222.9 | 175 | 0 | 60.9 |
| 18 | 13.855 | 226.0 | 150 | 0 | 51.5 |
| 19 | 15.070 | 227.5 | 125 | 0 | 42.6 |
| 20 | 16.560 | 227.5 | 100 | 0 | 34.1 |
| 21 | 18.055 | 228.1 | 80 | 0 | 27.2 |
| 22 | 18.950 | 228.4 | 70 | 0 | 23.8 |
| 23 | 19.986 | 228.1 | 60 | 0 | 20.4 |
| 24 | 21.203 | 227.0 | 50 | 0 | 17.1 |
| 25 | 22.697 | 228.9 | 40 | 0 | 13.6 |
| 26 | 24.650 | 231.2 | 30 | 0 | 10.1 |
| 27 | 25.895 | 232.1 | 25 | 0 | 8.4 |
| 28 | 27.411 | 232.3 | 20 | 0 | 6.7 |
| 29 | 29.400 | 234.4 | 15 | 0 | 5.0 |
| 30 | 32.215 | 237.4 | 10 | 0 | 3.3 |
| 31 | 34.720 | 240.6 | 7 | 0 | 2.3 |

TABLE XIV

Lihue, February 3, 1966

| Altitude Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0003210 | -0.00004288 | 0.000002127 |
| 9 - 18 | 0.0001008 | -0.00001007 | 0.000000256 |
| 18 - 27 | 0.0000291 | -0.00000447 | 0.000000229 |
| 27 - 36 | 0.0000065 | -0.00000095 | 0.000000050 |

TABLE XV

Lihue, July 2, 1966

| Altitude Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0003498 | -0.00005126 | 0.000002711 |
| 9 - 18 | 0.0001035 | -0.00001033 | 0.000000247 |
| 18 - 27 | 0.0000290 | -0.00000416 | 0.000000188 |
| 27 - 36 | 0.0000064 | -0.00000080 | 0.000000032 |

TABLE XVI

Fairbanks, February 3, 1966

| Altitude Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0003070 | -0.00003426 | 0.000001285 |
| 9 - 18 | 0.0000962 | -0.00001270 | 0.000000538 |
| 18 - 27 | 0.0000250 | -0.00000349 | 0.000000173 |
| 27 - 36 | 0.0000065 | -0.00000098 | 0.000000056 |

TABLE XVII

Fairbanks, July 2, 1966

| Altitude Range, km | | | |
|---|---|---|---|
| 0 - 9 | 0.0002844 | -0.00002865 | 0.000000950 |
| 9 - 18 | 0.0001063 | -0.00001392 | 0.000000569 |
| 18 - 27 | 0.0000267 | -0.00000349 | 0.000000148 |
| 27 - 36 | 0.0000071 | -0.00000094 | 0.000000040 |

TABLE XVIII

Lihue, February 3, 1966

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001855 | ±2.10 |
| 9 - 18 | 1.0000624 | ±0.12 |
| 18 - 27 | 1.0000152 | ±0.04 |
| 27 - 36 | 1.0000035 | ±0.01 |

TABLE XIX

Lihue, July 2, 1966

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001923 | ±2.10 |
| 9 - 18 | 1.0000637 | ±0.21 |
| 18 - 27 | 1.0000154 | ±0.06 |
| 27 - 36 | 1.0000036 | ±0.03 |

TABLE XX

Fairbanks, February 3, 1966

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001811 | ±0.31 |
| 9 - 18 | 1.0000590 | ±0.27 |
| 18 - 27 | 1.0000150 | ±0.00 |
| 27 - 36 | 1.0000042 | ±0.03 |

TABLE XXI

Fairbanks, July 2, 1966

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001811 | ±0.31 |
| 9 - 18 | 1.0000590 | ±0.27 |
| 18 - 27 | 1.0000150 | ±0.00 |
| 27 - 36 | 1.0000042 | ±0.03 |

## 5. Point-to-Point Determination of the Refractive Index for Light Waves

For light waves, the group refractive index is first computed by use of the Barrel and Sears [7] formula as follows:

$$(n_g - 1) \cdot 10^7 = A + \frac{3B}{\lambda^2} + \frac{5C}{\lambda^4} \tag{32}$$

where

$$\begin{aligned} A &= 2876.04 \\ B &= 16.288 \\ C &= 0.131 \end{aligned} \tag{33}$$

In the utilization of a ruby laser, the wavelength $\lambda = 6943$ Å is applied to Eq. (32) and yields the value of the group refractive index as:

$$(n_g - 1) \cdot 10^6 = 297.99 \tag{34}$$

TABLE XXII

Lihue, July 2 — Total Altitude Range 0.036 to 36.565 km — 35 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.019 | -0.005 | ±0.002 | +0.042 | ±0.000 | +0.009 | ±0.000 | ±0.000 | ±0.019 | +0.046 |
| 30° | 0.022 | -0.005 | 0.002 | +0.048 | 0.001 | +0.010 | 0.000 | 0.000 | 0.022 | +0.053 |
| 50° | 0.029 | -0.009 | 0.003 | +0.064 | 0.001 | +0.014 | 0.000 | 0.000 | 0.029 | +0.069 |
| 60° | 0.036 | -0.014 | 0.004 | +0.082 | 0.001 | +0.018 | 0.001 | 0.000 | 0.036 | +0.086 |
| 70° | 0.055 | -0.016 | 0.006 | +0.121 | 0.002 | +0.026 | 0.001 | 0.000 | 0.056 | +0.131 |

TABLE XXIII

Lihue, February 3 — Total Altitude Range 0.036 to 33.540 km — 34 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.019 | -0.067 | ±0.001 | +0.030 | ±0.000 | +0.007 | ±0.000 | -0.001 | ±0.019 | -0.031 |
| 30° | 0.022 | -0.077 | 0.001 | +0.034 | 0.000 | +0.008 | 0.000 | -0.001 | 0.022 | -0.036 |
| 50° | 0.029 | -0.104 | 0.002 | +0.046 | 0.001 | +0.011 | 0.000 | -0.002 | 0.029 | -0.049 |
| 60° | 0.038 | -0.133 | 0.002 | +0.059 | 0.001 | +0.014 | 0.000 | -0.002 | 0.038 | -0.062 |
| 70° | 0.055 | -0.195 | 0.003 | +0.089 | 0.002 | +0.021 | 0.000 | -0.003 | 0.055 | -0.088 |

TABLE XXIV

Fairbanks, July 2 — Total Altitude Range 0.499 to 34.720 km — 31 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.003 | -0.106 | ±0.003 | -0.001 | ±0.000 | +0.005 | ±0.000 | +0.005 | ±0.005 | -0.097 |
| 30° | 0.003 | -0.123 | 0.003 | -0.001 | 0.000 | +0.007 | 0.000 | +0.007 | 0.004 | -0.110 |
| 50° | 0.004 | -0.165 | 0.004 | -0.002 | 0.000 | +0.011 | 0.000 | +0.011 | 0.006 | -0.145 |
| 60° | 0.006 | -0.212 | 0.005 | -0.002 | 0.000 | +0.014 | 0.001 | +0.014 | 0.008 | -0.186 |
| 70° | 0.008 | -0.310 | 0.008 | -0.002 | 0.000 | +0.016 | 0.001 | +0.016 | 0.011 | -0.280 |

TABLE XXV

Fairbanks, February 3 — Total Altitude Range 0.146 to 31.362 km — 31 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.008 | -0.055 | ±0.003 | -0.052 | ±0.000 | -0.004 | ±0.000 | ±0.000 | ±0.009 | -0.111 |
| 30° | 0.010 | -0.063 | 0.003 | -0.060 | 0.000 | -0.004 | 0.001 | 0.000 | 0.011 | -0.127 |
| 50° | 0.013 | -0.085 | 0.004 | -0.081 | 0.001 | -0.006 | 0.001 | 0.000 | 0.014 | -0.172 |
| 60° | 0.016 | -0.110 | 0.005 | -0.104 | 0.001 | -0.007 | 0.002 | 0.000 | 0.017 | -0.221 |
| 70° | 0.024 | -0.160 | 0.008 | -0.152 | 0.001 | -0.011 | 0.002 | 0.000 | 0.026 | -0.323 |

To compute the ambient refractive refractive index for light waves, the Kohlrausch [7] formula is usually used, as follows:

$$N = (n - 1) \cdot 10^6 = \left[\frac{n_g - 1}{1 + \alpha\, t} \quad \frac{P}{760} - \frac{0.000000055 \cdot e}{1 + \alpha\, t}\right] 10^6 \qquad (35)$$

where P is the total pressure in millimeters of mercury, e is the partial pressure of water vapor in millimeters of mercury, $\alpha$ is the heat expansion coefficient of air and equals 0.00367, and t is the temperature in centigrade.

Since in actual soundings the total pressure is given in millibars, the author has modified Eq. (35) to be more suitable for computer programming in the following way [9]:

$$N = (n - 1) \quad 10^6 = \frac{298.0 \cdot P - 41.8 \cdot e}{3.709 \cdot T} \qquad (36)$$

where P and e are given in millibars, and T is the temperature in Kelvin units. In the derivation of Eq. (36) it was anticipated that $T_o = 273.15$, which yields the heat expansion coefficient $\alpha = 0.00366$.

## 5.1 Sensitivity of the Atmospheric Parameters

To investigate the sensitivity of atmospheric parameters when the light wave propagation is concerned, Eq. (36) must be differentiated as follows:

$$\begin{aligned}
dN_T &= \frac{1}{T^2}\left(-\frac{298.0}{3.709} P + \frac{41.8}{3.709} e\right) dT \\
dN_P &= \frac{298.0}{3.709 \cdot T} \cdot dP \qquad (37) \\
dN_e &= -\frac{41.8}{3.709 \cdot T} \cdot de
\end{aligned}$$

By the application of the T, P, and E values from (23) into (37) to correspond to an altitude of 4.5 km, the following relationships are obtained:

$$dN_T = 0.691 \cdot dt$$

$$dN_P = 0.310 \cdot dP \tag{38}$$

$$dN_e = -0.044 \cdot d_e$$

Again, if 1 ppm is set as reference criterion, the following standard errors of each parameter are valid:

$$m_T = \pm 1\overset{\circ}{.}4$$

$$m_P = \pm 3.2 \text{ mb} \tag{39}$$

$$m_e = \pm 22.9 \text{ mb}$$

On comparing values (39) to those of (24), it becomes very evident that the temperature and the pressure in both cases (propagation of radio and light waves) are of the same magnitude of sensitivity. While the humidity in the radio-wave propagation was the major factor in accuracy, in the light-wave propagation it is practically negligible. Even in hot and humid climate the partial pressure of water vapor at sea level rarely exceeds 23 mb, which means that Eq. (36) can be rewritten for the cool and dry climate as follows:

$$N = (n - 1) \cdot 10^6 = 80.35 \frac{P}{T} \tag{40}$$

## 6. Model of the Refractive Index of Light Waves from Standard Atmosphere

Similarly, as was done in the case of radio-wave propagation, the U. S. Standard Atmosphere was selected as the basis for the analysis. In Table XXVI, N-values obtained from Eq. (36) by the use of the T, P, and e-values from Table IV, are presented together with the altitudes.

The computed N-values were adjusted separately for the four altitude ranges, 0 to 9, 9 to 18, 18 to 27, and 27 to 36 kilometers. Curve-fitting adjustment of Eq. (12) yielded the following constants A, B, and C, given in Table XXVII.

The mean refractive indexes $\bar{n}$ and their standard errors in N-units for each altitude range are given in Table XXVIII. In Table XXIX, residuals after the adjustment are given at intervals of 1 km from 0 to 15 km, and at intervals of 5 km from 15 to 35 kilometers. L denotes the point-to-point value obtained from Eq. (36), P denotes the value read from the adjusted parabola, and $\Delta P$ denotes their difference.

The standard errors from Table XXVIII were converted in meters of distance corresponding to the zenith angles of $\psi$ = 70°, 60°, 50°, 30°, and 0°. These, together with the cumulative standard errors from sea level to 36 kilometers are given in Table XXX.

The largest standard error at the poorest zenith angle, $\psi$ = 70°, and through the altitude range, 0 to 9 km, is only ± 1 millimeter. This accuracy is five times better than that obtained by the use of radio waves through the same altitude range. Through the altitude range of 9 to 18 km, standard errors in both cases are ± 3 millimeters. This comparison justifies the use of the modified formula (40), even in the lower atmosphere where moisture is present. Tables XXVI - XXX follow.

TABLE XXVI

| No. | Geom. Alt., km | N | No. | Geom. Alt., km | N |
|---|---|---|---|---|---|
| 1 | 0 | 282.0 | 38 | 18.5 | 26.0 |
| 2 | 0.5 | 268.9 | 39 | 19.0 | 24.1 |
| 3 | 1.0 | 256.1 | 40 | 19.5 | 22.2 |

TABLE XXVI (continued)

| | | | | | |
|---|---|---|---|---|---|
| 4 | 1.5 | 243.9 | 41 | 20.0 | 20.4 |
| 5 | 2.0 | 231.9 | 42 | 20.5 | 18.9 |
| 6 | 2.5 | 220.5 | 43 | 21.0 | 17.4 |
| 7 | 3.0 | 209.4 | 44 | 21.5 | 16.2 |
| 8 | 3.5 | 199.1 | 45 | 22.0 | 14.7 |
| 9 | 4.0 | 189.0 | 46 | 22.5 | 13.6 |
| 10 | 4.5 | 179.0 | 47 | 23.0 | 12.8 |
| 11 | 5.0 | 169.6 | 48 | 23.5 | 11.7 |
| 12 | 5.5 | 160.7 | 49 | 24.0 | 10.9 |
| 13 | 6.0 | 152.2 | 50 | 24.5 | 10.2 |
| 14 | 6.5 | 144.1 | 51 | 25.0 | 9.1 |
| 15 | 7.0 | 136.1 | 52 | 25.5 | 8.3 |
| 16 | 7.5 | 128.5 | 53 | 26.0 | 7.9 |
| 17 | 8.0 | 121.4 | 54 | 26.5 | 7.2 |
| 18 | 8.5 | 114.5 | 55 | 27.0 | 6.8 |
| 19 | 9.0 | 107.7 | 56 | 27.5 | 6.1 |
| 20 | 9.5 | 101.5 | 57 | 28.0 | 5.7 |
| 21 | 10.0 | 95.3 | 58 | 28.5 | 5.4 |
| 22 | 10.5 | 89.5 | 59 | 29.0 | 5.0 |
| 23 | 11.0 | 84.1 | 60 | 29.5 | 4.6 |
| 24 | 11.5 | 77.9 | 61 | 30.0 | 4.3 |
| 25 | 12.0 | 71.9 | 62 | 30.5 | 3.9 |
| 26 | 12.5 | 66.4 | 63 | 31.0 | 3.6 |
| 27 | 13.0 | 61.5 | 64 | 31.5 | 3.4 |
| 28 | 13.5 | 56.7 | 65 | 32.0 | 3.1 |
| 29 | 14.0 | 52.6 | 66 | 32.5 | 2.9 |
| 30 | 14.5 | 48.6 | 67 | 33.0 | 2.7 |
| 31 | 15.0 | 44.9 | 68 | 33.5 | 2.5 |
| 32 | 15.5 | 41.5 | 69 | 34.0 | 2.3 |
| 33 | 16.0 | 38.6 | 70 | 34.5 | 2.1 |
| 34 | 16.5 | 35.6 | 71 | 35.0 | 1.9 |
| 35 | 17.0 | 32.6 | 72 | 35.5 | 1.8 |
| 36 | 17.5 | 30.4 | 73 | 36.0 | 1.7 |
| 37 | 18.0 | 28.2 | 74 | 36.5 | 1.5 |

TABLE XXVII

| Altitude Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0002818 | -0.000026408 | 0.0000007964 |
| 9 - 18 | 0.0001085 | -0.000013852 | 0.0000005465 |
| 18 - 27 | 0.0000279 | -0.000003998 | 0.0000001877 |
| 27 - 36 | 0.0000066 | -0.000000907 | 0.0000000400 |

TABLE XXVIII

| Altitude Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001845 | ±0.04 |
| 9 - 18 | 1.0000610 | ±0.11 |
| 18 - 27 | 1.0000150 | ±0.05 |
| 27 - 36 | 1.0000036 | ±0.02 |

TABLE XXIX

| Altitude, km | L, N-Value | P, N-Value | ΔP, N-Value | Altitude, km | L, N-Value | P, N-Value | ΔP, N-Value |
|---|---|---|---|---|---|---|---|
| 0 | 282.0 | 281.8 | +0.2 | 10 | 95.3 | 95.1 | +0.2 |
| 1 | 256.1 | 256.2 | -0.1 | 11 | 84.1 | 83.0 | +1.1 |
| 2 | 231.9 | 232.2 | -0.3 | 12 | 71.9 | 71.8 | +0.1 |
| 3 | 209.4 | 209.8 | -0.4 | 13 | 61.5 | 61.9 | -0.4 |
| 4 | 189.0 | 188.9 | +0.1 | 14 | 52.6 | 52.9 | -0.3 |
| 5 | 169.6 | 169.6 | ±0 | 15 | 44.9 | 45.1 | -0.2 |

TABLE XXIX (continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 152.2 | 152.0 | +0.2 | 20 | 20.4 | 20.7 | -0.3 |
| 7 | 136.1 | 135.9 | +0.2 | 25 | 9.1 | 9.1 | ±0 |
| 8 | 121.4 | 121.4 | ±0 | 30 | 4.3 | 4.3 | ±0 |
| 9 | 107.7 | 108.5 | -0.8 | 35 | 1.9 | 1.9 | ±0 |

TABLE XXX

| Altitude Range, km | Zenith Angle | | | | |
|---|---|---|---|---|---|
| | 70° | 60° | 50° | 30° | 0° |
| 0 - 9 | ±0.001 | ±0.001 | ±0.001 | ±0.000 | ±0.000 |
| 9 - 18 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 |
| 18 - 27 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 |
| 27 - 36 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0 - 36 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 |

## 7. Model of the Refractive Index of Light Waves from Actual Meteorological Soundings

Similarly with the case of the use of radio waves, actual meteorological soundings made in Lihue, Hawaii and Fairbanks, Alaska, February 3, 1966 and July 2, 1966, were used in the analysis of the light waves.

Actual soundings with the T, P, and e values from tables X - XIII were applied to Eq. (36) to obtain the N-values and are presented in tables XXXI - XXXIV. Curve-fitting adjustment of the N-values with the parabola (12) yielded the following constants A, B, and C given in tables XXXV - XXXVIII. The mean refractive indexes through the four altitude ranges and their standard errors in N-units are presented in tables XXXIX - XLII. These standard errors were converted to correspond to standard errors in distances, in meters, for the previously used zenith angles and altitude ranges. Also, the constant shift between the use of the U. S. Standard Atmosphere and the local, actual atmosphere was computed. The results are given in tables XLIII - XLVI. Tables XXXI - XLVI follow.

TABLE XXXI

Lihue, Hawaii, February 3, 1966

| No. | Geom. alt., km | N | No. | Geom. alt., km | N |
|---|---|---|---|---|---|
| 1 | 0.036 | 274.5 | 18 | 12.335 | 72.2 |
| 2 | 0.114 | 272.9 | 19 | 13.202 | 64.9 |
| 3 | 0.552 | 263.1 | 20 | 14.167 | 57.5 |
| 4 | 1.012 | 253.1 | 21 | 15.273 | 49.7 |
| 5 | 1.490 | 240.6 | 22 | 16.580 | 40.6 |
| 6 | 1.994 | 227.1 | 23 | 17.884 | 32.5 |
| 7 | 2.505 | 214.2 | 24 | 18.664 | 28.3 |
| 8 | 3.099 | 201.5 | 25 | 19.582 | 23.4 |
| 10 | 4.348 | 176.7 | 27 | 22.083 | 15.2 |
| 11 | 5.031 | 164.9 | 28 | 23.915 | 11.1 |
| 12 | 5.785 | 153.3 | 29 | 25.130 | 9.1 |
| 13 | 6.575 | 140.7 | 30 | 26.450 | 7.2 |
| 14 | 7.472 | 127.7 | 31 | 28.442 | 5.4 |
| 15 | 8.451 | 113.4 | 32 | 31.137 | 3.5 |
| 16 | 9.564 | 99.5 | 33 | 33.540 | 2.4 |
| 17 | 10.839 | 85.8 | | | |

TABLE XXXII

Lihue, Hawaii, July 2, 1966

| No. | Geom. Alt., km | N | No. | Geom. Alt., km | N |
|---|---|---|---|---|---|
| 1 | 0.036 | 270.4 | 18 | 12.447 | 73.0 |
| 2 | 0.150 | 270.1 | 19 | 13.300 | 66.2 |

TABLE XXXII (continued)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0.591 | 259.9 | 20 | 14.253 | 53.4 |
| 4 | 1.062 | 249.5 | 21 | 15.344 | 49.5 |
| 5 | 1.547 | 237.9 | 22 | 16.689 | 39.5 |
| 6 | 2.055 | 225.5 | 23 | 18.040 | 31.0 |
| 7 | 2.594 | 213.6 | 24 | 18.855 | 26.9 |
| 8 | 3.170 | 197.9 | 25 | 19.805 | 22.9 |
| 9 | 3.777 | 185.2 | 26 | 20.940 | 18.9 |
| 10 | 4.446 | 173.9 | 27 | 22.347 | 14.9 |
| 11 | 5.131 | 162.4 | 28 | 24.202 | 10.9 |
| 12 | 5.903 | 150.8 | 29 | 25.406 | 8.9 |
| 13 | 6.717 | 139.0 | 30 | 26.900 | 7.0 |
| 14 | 7.612 | 126.2 | 31 | 28.850 | 5.2 |
| 15 | 8.597 | 113.8 | 32 | 31.642 | 3.4 |
| 16 | 9.699 | 101.0 | 33 | 34.165 | 2.4 |
| 17 | 10.960 | 86.9 | 34 | 36.565 | 1.7 |

TABLE XXXIII

Fairbanks, Alaska, February 3, 1966

| No. | Geom. Alt., km | N | No. | Geom. Alt., km | N |
|---|---|---|---|---|---|
| 1 | 0.146 | 316.1 | 18 | 12.240 | 62.5 |
| 2 | 0.539 | 286.2 | 19 | 13.261 | 52.9 |
| 3 | 0.960 | 269.8 | 20 | 14.480 | 44.3 |
| 4 | 1.406 | 257.1 | 21 | 15.964 | 35.5 |
| 5 | 1.877 | 244.1 | 22 | 17.442 | 28.5 |
| 6 | 2.376 | 231.8 | 23 | 18.324 | 24.9 |
| 7 | 2.893 | 220.3 | 24 | 19.346 | 21.4 |
| 8 | 3.446 | 208.2 | 25 | 20.556 | 17.8 |
| 9 | 4.026 | 194.7 | 26 | 22.030 | 14.2 |
| 10 | 4.658 | 180.7 | 27 | 23.955 | 10.6 |

TABLE XXXIII

| | | | | | |
|---|---|---|---|---|---|
| 11 | 5.331 | 167.1 | 28 | 25.173 | 8.5 |
| 12 | 6.056 | 153.7 | 29 | 26.672 | 6.8 |
| 13 | 6.871 | 139.7 | 30 | 28.612 | 5.1 |
| 14 | 7.759 | 125.2 | 31 | 31.362 | 3.3 |
| 15 | 8.758 | 110.4 | | | |
| 16 | 9.917 | 92.3 | | | |
| 17 | 11.365 | 72.0 | | | |

TABLE XXXIV

Fairbanks, Alaska, July 2, 1966

| No. | Geom. Alt., km | N | No. | Geom. Alt., km | N |
|---|---|---|---|---|---|
| 1 | 0.499 | 257.2 | 17 | 12.837 | 63.1 |
| 2 | 0.961 | 247.8 | 18 | 13.855 | 53.3 |
| 3 | 1.445 | 238.3 | 19 | 15.070 | 44.1 |
| 4 | 1.947 | 228.4 | 20 | 16.560 | 35.3 |
| 5 | 2.464 | 218.2 | 21 | 18.055 | 28.2 |
| 6 | 3.027 | 206.9 | 22 | 18.950 | 24.6 |
| 7 | 3.615 | 194.1 | 23 | 19.986 | 21.1 |
| 8 | 4.242 | 182.5 | 24 | 21.203 | 17.7 |
| 9 | 4.900 | 171.1 | 25 | 22.697 | 14.0 |
| 10 | 5.626 | 158.3 | 26 | 24.650 | 10.4 |
| 11 | 6.390 | 143.7 | 27 | 25.895 | 8.6 |
| 12 | 7.263 | 130.5 | 28 | 27.411 | 6.9 |
| 13 | 8.221 | 116.6 | 29 | 29.400 | 5.1 |
| 14 | 9.295 | 103.0 | 30 | 32.215 | 3.4 |
| 15 | 10.520 | 89.3 | 31 | 34.720 | 2.3 |
| 16 | 11.973 | 73.4 | | | |

TABLE XXXV

Lihue, February 3, 1966

| Alt. Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0002767 | -0.00002666 | 0.000000883 |
| 9 - 18 | 0.0001049 | -0.00001070 | 0.000000288 |
| 18 - 27 | 0.0000300 | -0.00000456 | 0.000000230 |
| 27 - 36 | 0.0000067 | -0.00000095 | 0.000000046 |

TABLE XXXVI

Lihue, July 2, 1966

| Alt. Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0002744 | -0.00002632 | 0.000000893 |
| 9 - 18 | 0.0001080 | -0.00001096 | 0.000000270 |
| 18 - 27 | 0.0000304 | -0.00000444 | 0.000000206 |
| 27 - 36 | 0.0000066 | -0.00000085 | 0.000000035 |

TABLE XXXVII

Fairbanks, February 3, 1966

| Alt. Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0003071 | -0.00003378 | 0.000001335 |
| 9 - 18 | 0.0001050 | -0.00001540 | 0.000000760 |
| 18 - 27 | 0.0000261 | -0.00000370 | 0.000000185 |
| 27 - 36 | 0.0000067 | -0.00000101 | 0.000000063 |

TABLE XXXVIII

Fairbanks, July 2, 1966

| Alt. Range, km | A | B | C |
|---|---|---|---|
| 0 - 9 | 0.0002689 | -0.00002199 | 0.000000408 |
| 9 - 18 | 0.0001079 | -0.00001365 | 0.000000532 |
| 18 - 27 | 0.0000279 | -0.00000372 | 0.000000162 |
| 27 - 36 | 0.0000073 | -0.00000098 | 0.000000044 |

TABLE XXXIX

Lihue, February 3, 1966

| Alt. Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001806 | ±0.37 |
| 9 - 18 | 1.0000645 | ±0.15 |
| 18 - 27 | 1.0000157 | ±0.03 |
| 27 - 36 | 1.0000037 | ±0.01 |

TABLE XL

Lihue, July 2, 1966

| Alt. Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001801 | ±0.49 |
| 9 - 18 | 1.0000660 | ±0.20 |
| 18 - 27 | 1.0000160 | ±0.08 |
| 27 - 36 | 1.0000037 | ±0.04 |

TABLE XLI

Fairbanks, February 3, 1966

| Alt. Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001911 | ±1.52 |
| 9 - 18 | 1.0000562 | ±0.39 |
| 18 - 27 | 1.0000144 | ±0.03 |
| 27 - 36 | 1.0000038 | ±0.14 |

TABLE XLII

Fairbanks, July 2, 1966

| Alt. Range, km | $\bar{n}$ | $m_{\bar{N}}$ |
|---|---|---|
| 0 - 9 | 1.0001810 | ±0.25 |
| 9 - 18 | 1.0000608 | ±0.35 |
| 18 - 27 | 1.0000155 | ±0.03 |
| 27 - 36 | 1.0000041 | ±0.05 |

## 8. Conclusion and Recommendations

Due to the limited number of samples analyzed, no final conclusion can be drawn. However, a very clear correlation exists between standard errors obtained in a hot and humid climate (Hawaii) and in a cold and dry climate (Alaska) when radio-wave propagation is utilized. The total standard errors from sea level to 36 kilometers were ±6 cm and ±6 cm, based on soundings made in Hawaii during the months of July and February, and ±1 cm and ±3 cm, based on soundings made in Alaska during the months of February and July.

TABLE XLIII

Lihue, July 2 | Total Altitude Range 0.036 to 36.565 km | 35 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.004 | -0.040 | ±0.002 | +0.045 | ±0.001 | +0.009 | ±0.000 | +0.001 | ±0.005 | +0.015 |
| 30° | 0.005 | -0.046 | 0.002 | +0.052 | 0.001 | +0.010 | 0.000 | +0.001 | 0.005 | +0.017 |
| 50° | 0.007 | -0.062 | 0.003 | +0.070 | 0.001 | +0.014 | 0.001 | +0.001 | 0.008 | +0.023 |
| 60° | 0.009 | -0.079 | 0.004 | +0.090 | 0.001 | +0.018 | 0.001 | +0.002 | 0.010 | +0.031 |
| 70° | 0.013 | -0.116 | 0.005 | +0.132 | 0.002 | +0.026 | 0.001 | +0.003 | 0.014 | +0.045 |

TABLE XLIV

Lihue, February 3 | Total Altitude Range 0.036 to 33.540 km | 34 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.003 | -0.035 | ±0.001 | +0.031 | ±0.000 | +0.006 | ±0.000 | +0.001 | ±0.003 | +0.003 |
| 30° | 0.004 | -0.041 | 0.002 | +0.036 | 0.000 | +0.007 | 0.000 | +0.001 | 0.004 | +0.003 |
| 50° | 0.005 | -0.055 | 0.002 | +0.049 | 0.000 | +0.010 | 0.000 | +0.001 | 0.005 | +0.005 |
| 60° | 0.007 | -0.070 | 0.003 | +0.063 | 0.001 | +0.013 | 0.000 | +0.002 | 0.008 | +0.008 |
| 70° | 0.010 | -0.103 | 0.004 | +0.092 | 0.001 | +0.018 | 0.000 | +0.003 | 0.011 | +0.010 |

TABLE XLV

Fairbanks, July 2 — Total Altitude Range 0.499 to 34.720 km — 31 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.002 | −0.032 | ±0.003 | −0.002 | ±0.000 | +0.005 | ±0.000 | +0.005 | ±0.004 | −0.024 |
| 30° | 0.003 | −0.036 | 0.004 | −0.002 | 0.000 | +0.005 | 0.001 | +0.005 | 0.005 | −0.028 |
| 50° | 0.004 | −0.049 | 0.005 | −0.003 | 0.000 | +0.007 | 0.001 | +0.007 | 0.007 | −0.038 |
| 60° | 0.005 | −0.063 | 0.006 | −0.004 | 0.001 | +0.009 | 0.001 | +0.009 | 0.008 | −0.049 |
| 70° | 0.007 | −0.092 | 0.009 | −0.005 | 0.001 | +0.013 | 0.001 | +0.013 | 0.012 | −0.071 |

TABLE XLVI

Fairbanks, February 3 — Total Altitude Range 0.146 to 31.362 km — 31 Soundings

| | 0 to 9 km | | 9 to 18 km | | 18 to 27 km | | 27 to 36 km | | 0 to 36 km | |
|---|---|---|---|---|---|---|---|---|---|---|
| Zenith Angle | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m | St. Error, m | Differ., m |
| 0° | ±0.014 | +0.059 | ±0.004 | −0.043 | ±0.000 | −0.006 | ±0.001 | +0.002 | ±0.015 | +0.012 |
| 30° | 0.015 | +0.069 | 0.004 | −0.050 | 0.000 | −0.006 | 0.002 | +0.002 | 0.016 | +0.015 |
| 50° | 0.021 | +0.092 | 0.005 | −0.067 | 0.000 | −0.008 | 0.002 | +0.003 | 0.022 | +0.020 |
| 60° | 0.027 | +0.119 | 0.007 | −0.086 | 0.001 | −0.011 | 0.003 | +0.004 | 0.028 | +0.026 |
| 70 | 0.040 | +0.173 | 0.010 | −0.124 | 0.001 | −0.016 | 0.004 | +0.005 | 0.042 | +0.038 |

The largest difference between distances based on standard-atmosphere data and local-atmosphere data (Fairbanks, February 3, 1966) was -32 cm, which shows that to achieve the ultimate accuracy in utilizing the radio-wave propagation, soundings should be used that have been made in the vicinity or proximity of the tracking station.

In the case of laser and, consequently, light-wave propagation, the influence of humidity is practically negligible. Therefore, standard errors obtained in measurements based on either humid- or dry-climate soundings are of the same order. Due to the absence of the humidity effect, the accuracy of laser tracking is significantly higher than tracking by radio waves. At the zenith angle $\psi = 70°$, and through the altitude range of 0 to 9 km, the standard error of the mean refractive index based on the U. S. Standard Atmosphere is only ±1 mm, compared to ±5 mm when the radio waves are used. The largest standard error for the total altitude range from 0 to 36 km (Fairbanks, February 3, 1966), was ±4.2 cm, while the difference between distances based on the standard and local atmospheres was +3.8 cm. This suggests that properly selected and computed standard atmosphere values guarantee sufficient accuracy to most geodetic and geophysical applications when light waves are utilized.

In the continuation of this research, emphasis should be put to investigate the influence of the location of the tracking station to the surrounding sounding stations by a proper interpolation of the data area-wise. Also the interpolation time-wise between the 00.00 and 12.00 Greenwich-time soundings should be made, together with a proper error analysis. The results of this analysis should then be applied to radio-wave tracking.

In the case of the light-wave tracking, an empirical standard atmosphere should be built up based on a great number of sounding values covering large areas of national and international tracking sites both for daytime and nighttime. The parabolic constants through four different altitude ranges of this adjusted standard atmosphere should be used and the standard errors analyzed.

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